

The Contamination Problem in Energy Regulation[#]

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Abstract: This paper formally examines the implications of an electricity utility's diversification into an unregulated industry. In our framework, the electricity utility is the most efficient provider in the unregulated industry (up to a particular capacity) and, as such, there is no question about the desirability of allowing it to operate in that market. Nevertheless, the risk faced by a diversified utility is greater than the risk faced by a utility that operates only in a regulated market. This additional risk can potentially affect the diversified utility's credit rating and, therefore, increase the cost of capital for the regulated business that will be recovered from ratepayers. We show that by allowing a regulated firm to diversify into an unregulated market, the energy regulator faces a trade-off: a lower cost in the unregulated market versus a higher cost in the regulated market. If the regulator only cares about welfare in the regulated market, then a ring-fencing requirement is optimal subject to implementation costs not being substantial. Of course, the ring-fencing requirement effectively prevents the firm from achieving a lower cost in the unregulated market. Therefore, if the regulator cares about welfare in both regulated and unregulated markets, ring-fencing may no longer be optimal.

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1. Introduction

The landscape in which energy utilities operate has changed dramatically over the past two decades. Monolithic structures which were previously publicly owned have been vertically separated, privatized or given strictly commercial, profit-maximizing mandates. The energy corporations that have emerged are often diversified and exhibit sophisticated corporate structures. In particular, a structure that appears worldwide is that of a holding company which has ownership interests across a number of activities where not all those activities are regulated.

Examples are plentiful: UK-based Centrica operates in gas and electricity transmission, in competitive segments of energy sectors as well as in drain and plumbing services; France's Suez-Lyonnaise des Eaux, Germany's RWE and Italy's Enel operate across regulated and unregulated markets in energy and other utility sectors. AGL is one of Australia's leading integrated energy companies with interests that include retail and merchant energy businesses, power generation assets and an upstream gas portfolio. In the United States, one third of over two hundred existing electricity retailers such as Northern States Power and Potomac Electric and Power Company also offer telecommunication services.

Although in many of the examples above diversification by an electricity utility resulted in corporate structures with ownership interests in more than one utility sector (and potentially subject to price regulation across different sectors), the focus of this paper is on diversification into unregulated (and potentially unrelated) markets. Examples of this type of diversification by an electricity utility include the leasing by Potomac Electric Power of Boeing 747s to KLM and Singapore Airlines; the acquisition by FPL Group (formerly Florida Power & Light) of the insurance company Colonial Penn Group; the purchase by Pacific Lighting Corp of a chain of drug stores and the acquisition by Pinnacle West (the parent company of Arizona Public Service) of Merabank, Arizona's largest savings and loan association.¹

The existing economics literature on the diversification of regulated firms which is reviewed in the next section, focuses on the firm's incentive or ability to transfer costs from the unregulated to the regulated business. The standard concern within the literature highlights the need for increased regulatory oversight post diversification to prevent the regulated business claiming higher regulated prices and competing unfairly in the unregulated market.

¹ For example; Jandik and Makhija (2005).

This paper, however, examines the problem of contamination, which emerges in its simplest form when an electricity utility that operates only in a regulated market invests in an unregulated market affecting its own credit rating. In our framework and given that the utility is the most efficient provider of the unregulated business (up to a particular capacity), it is desirable that it be allowed to operate in that market. However, as the risk faced by a diversified utility is greater than the risk faced by a utility that does not diversify, it is likely that the utility's credit rating will be affected. Consequently, *ceteris paribus*, the cost of debt for a utility that operates only in a regulated market is lower than for a diversified utility that operates in an unregulated market as well. This increases the cost of capital for the regulated business that will be recovered from ratepayers. We show that by allowing a regulated firm to diversify into an unregulated market the energy regulator faces a trade-off: a lower cost in the unregulated market versus a higher cost in the regulated market. If the regulator only cares about welfare in the regulated market, then a ring-fencing requirement is optimal, subject to implementation costs being insubstantial. As the ring-fencing requirement prevents the firm from achieving a lower cost in the unregulated market, under certain conditions ring-fencing will not be optimal for a regulator that is concerned about welfare in both markets. A distinction from earlier literature is that it is the market that provides the incentive for firm cross-subsidisation. Cross-subsidisation is not a strategic choice for the regulated firm.

This paper is organised as follows: Section 2 reviews the existing literature on diversification by utilities; Section 3 discusses some of the institutional features of the interaction between the existence of ring-fencing, which aims at isolating risk, and credit rating; Section 4 introduces the model; Section 5 investigates when it is socially optimal to ring-fence the regulated business in order to avoid the contamination problem, and Section 6 concludes the paper.

2. The Existing Literature on Diversification

There are many explanations of why regulated firms have been allowed to expand and diversify into unregulated markets. Lewis and Sappington (1989) develop a model in which earnings in the unregulated market are positively correlated with costs in the regulated market. In this setting, the firm's incentive to exaggerate production costs in the regulated market is mitigated because such an exaggeration implies a claim that participation in the unregulated market is more profitable than it actually is. The regulator's task of controlling cost exaggeration in the regulated market can actually be made less burdensome by allowing participation in unregulated markets. Given

this, a regulator may be able to enhance the level of expected consumer surplus in a regulated market by allowing the regulated firm to enter unregulated markets.

Palmer (1991) investigates the implications of a firm's decision to diversify into an unregulated market for both a firm's incentive to innovate and for the level of consumer welfare. The technology is assumed to exhibit economies of scale in the production of the regulated product as well as economies of scope in the joint production of the two products when the cost function for the regulated product has both fixed and variable components. Investment in R&D reduces both components through an innovation factor. If the firm seeks to diversify into the production of the unregulated product, it must obtain the approval of the regulator who may simultaneously lower the regulated revenue requirement and the regulated price in this period. This is accomplished by shifting the allocation of a fraction θ of the fixed cost of producing the regulated product to the unregulated product.

The regulator chooses a value of θ that reflects his attitudes towards how the benefits of economies of scope should be split between the regulated and unregulated markets. Under this rule, the firm diversifies if θ is below the firm's breakeven level, θ^* . The diversified firm chooses a higher level of R&D investment and consumers of the regulated product are better off if the regulated firm diversifies because of the lower average revenue requirement. Over time, the policy of sharing fixed costs will lead to higher consumer welfare gains even if the diversified firm's R&D investment is biased toward fixed-cost-reducing R&D.

Palmer's analysis assumes that the regulator is able to distinguish joint and attributable costs and accurately assess, *ex post*, the effects of R&D investment on the firm's cost function. If the regulator is unable to categorize costs or to observe cost reductions, then the negative effects of cross subsidisation, distorted technology choice (or both), may outweigh any long-run benefits to consumers from reduced costs.

Another stream of literature investigates the potential negative consequences that result from the ability of the regulated firm to shift costs from the unregulated market to the regulated market. Braeutigam and Panzar (1989), for example, study the incentives for firms to shift costs from competitive to monopoly markets across two types of regulatory policies: rate-of-return and price-cap regulation. They conclude that rate-of-return regulation provides the firm with incentive to misreport cost allocations and in some cases to choose an inefficient technology, undertake cost-reducing innovation in an inefficient way, under-produce and price below cost in the competitive market. In contrast, pure price-cap regulation can induce the firm to minimize costs, produce efficiently in non-core markets, undertake cost-reducing innovation as an unregulated firm would,

and diversify into a non-core market if and only if diversification is efficient. There is then no incentive to misreport cost allocations and choose an inefficient technology since cost allocation is not required under this regulatory scheme.

Brennan (1990) analyses two cross-subsidization tactics – costs misallocation and distorted technological choice – under a spectrum of regulatory cost allocation policies. In his model, output in the regulated market will be reduced as higher prices are set by the regulator to fund the cross-subsidy. In the unregulated market, the price may fall below true marginal production cost and efficient production by others may be displaced. Moreover, if competitors in the unregulated market are large and thus render game theoretic considerations as more important, the potential for cross-subsidization may enable a regulated firm to maintain a monopoly price in an unregulated market. However, where there are economies of scope, increasing the regulated firm's share in the unregulated market at the expense of its competitors may increase welfare. The marginal inefficiency from overproduction of the competitive good by a regulated firm or over-utilization of the regulated good by a vertically related firm, may be outweighed by the marginal gain when such production generates economies of scope that benefit ratepayers.

Brennan and Palmer (1994) identify conditions under which gains from economies of scope and increased competition in unregulated markets tend to outweigh the costs of cross-subsidization. They use a perfect competition model of the unregulated market to examine trade-offs under economies of scope. Effects of increased competition are assessed using Cournot models with linear and constant elasticity demands. They show that diversification trade-offs depend on variables that regulators should be able to estimate. For instance, the likelihood of a welfare loss rises as the elasticity of demand, level of competition in the unregulated market, or the size of the cross-subsidy grows.

Brennan and Palmer show that, with no fixed costs, diversification is beneficial when it leads to net entry. However, diversification in the presence of fixed costs leads to welfare gains only if the unregulated market is highly concentrated and demand is weakly inelastic. When there are economies of scope, cross-subsidized diversification is more likely to enhance welfare the larger the economies of scope, the smaller the cross-subsidy, or the smaller the firm's elasticity of supply for the unregulated product. Diversification can reduce welfare even without a loss in the regulated market if cross-subsidy induces inefficient production and economies of scope are not extensive. Economies of scope appear to lend more support for diversification than does increased competition.

More recent literature focuses on the optimal design of regulation to account for the possibility of diversification into unregulated markets. Sappington (2003) develops a model where effort can be allocated to regulated and unregulated activities. In this framework, horizontal diversification can provide important benefits for customers of the firm's core regulated service. In particular, some of the profit the firm earns in non-core markets can be taxed (explicitly or implicitly) and delivered to core customers. However, diversification may harm core customers if the prospect of significant profit in non-core markets induces the regulated firm to divert its creative energies from its core market to non-core markets.

Sappington investigates this trade off and concludes that diversification would be advantageous for core customers if the effort diversion problem were not present and if the regulator were well informed about operations in non-core markets (and so able to distinguish between necessary and unnecessary expenditures by the regulated firm in these markets). Under these conditions, a regulator who is capable of perfectly monitoring the firm's allocation of cost-reducing effort across markets would always authorize diversification.

In contrast, a combination of effort diversion and cost padding problems may lead the regulator to prohibit diversification. Moreover, a regulator is more likely to prohibit diversification when its knowledge of non-core markets is limited, when the incremental surplus that the regulated firm is certain to secure in non-core markets is small, or when the potential cost variation in the core market is large (in this case, the gains from reducing costs in the core market are large).

In addition, the regulator may authorize diversification into a given non-core market when the impact of the firm's cost-reducing effort in the core market (and thus the potential loss from effort diversion) is substantial, but preclude diversification when the impact is more limited. Whenever the regulator authorizes diversification, he admits profit in the core market that increases as realized operating costs decline. Doing so limits the incentive of the regulated firm to divert effort from its core operations to its non-core operations.

Finally, Calzolari and Scarpa (2007) study the regulation of a utility firm which, when it jointly operates in competitive and unregulated sectors, enjoys economies of scope. The size of scope economies is assumed to be private information. These authors show that if scope economies are significant, consumers in the two markets may benefit from the efficiency gain of integrated production. The lack of information on the part of the regulator and the behaviour of the rival firms in the competitive sector distort the price in the regulated market and may negatively impact on competition in the unregulated market. Nevertheless, on balance, the authors show that allowing the utility to operate in the competitive market is desirable.

While the above literature focuses by and large on the incentives for and consequences of cost misallocation between unregulated and regulated markets, this paper focuses instead on the contamination problem. In order to understand how contamination might come about, it is useful to review the current credit agencies' practices when rating electricity utilities. This is the subject of the next Section.

3. Ring-Fencing and Credit Rating

Ring-fencing generally involves techniques to insulate the credit risk of an issuer from the risks of affiliate issuers within a corporate structure.² These techniques include, among others: capital structure requirements; dividend restrictions; unregulated investment restrictions; prohibitions on utility asset sales; collateralization requirements; working capital restrictions; prohibitions on inter-company loans; maintenance of stand-alone bonds and independence of board members.³

The relationship between ring-fencing and credit rating is explicitly considered by credit rating agencies. For example, Moody's rating methodology for regulated electric utilities comprises assessment of five elements: (i) the extent of a regulated company's exposure to its unregulated businesses – the strongest credit risk position is enjoyed by a company whose business is wholly regulated and where non-utility activities are substantial, the main credit driver will be the assessment of these businesses; (ii) the credit support that is gained from operating within a particular regulatory framework, including ring-fencing mechanisms; (iii) the exact level of risk posed by the unregulated businesses to the overall credit; (iv) six specific financial ratios which are considered the most useful when assessing an electric utility and the adjustments made to calculate these, and (v) more generic risk factors that are not specific to utility companies, e.g. the adequacy of liquidity arrangements and the appetite for acquisitions.⁴

Moody's methodology includes an assessment of the operations of the unregulated division in order to determine the overall rating of an electricity utility. In particular, consideration is given to the extent of regulatory imposed ring-fencing restrictions on dividends, capital expenditures (CAPEX) and investments, separate financings, separate legal structure and limits on the ability of the regulated entity to support its parent or affiliate company.

² See, for example, Bonelli et al (2003).

³ See, for example, Regulatory Research Associates (2003) and Beach et al (2005).

⁴ See Moody's (2005).

Standard & Poor's rating methodology explicitly states that the agency would rarely view the default risk of a regulated subsidiary as being substantially different from the credit quality of the consolidated economic entity, which would fully take into account the parent company's obligations. Regulated subsidiaries can only be treated as exceptions to this rule if there are substantive ring-fencing mechanisms. In this case, the rating of the regulated subsidiary would be more reflective of its 'stand-alone' credit profile. As a corollary, the parent and affiliates rating is negatively affected in the presence of ring-fencing since they are deprived of full access to the subsidiary's assets and cash flow.⁵

Finally, Fitch's rating methodology states that it will generally undertake separate analysis of the respective probabilities of default when cash flows between a holding company and its subsidiaries are formally restricted via ring-fencing mechanisms. This will result in the assignment of different ratings when the two entities have different stand-alone credit profiles. Such ring-fencing mechanisms can take the form of restricted dividend covenants or minimum financial ratios before paying subordinated inter-company loan obligations.⁶

The methodologies of the credit rating agencies discussed above suggest that in the absence of ring-fencing mechanisms, the rating of an electricity utility is obtained by the aggregation of the risks facing both the regulated and unregulated divisions. This leads to a regulated division being penalized via a lower credit rating.

The rationale for the importance given to ring-fencing mechanisms by the credit agencies' methodologies relates to the possible financial losses that the regulated utility can suffer as a result of its linkage to the unregulated parent or affiliate companies. Burns and Murphy (2006) point out that these losses might occur when utility assets or revenue streams are used as collateral for upstream or affiliate loans, or when utility dividends or working capital are moved elsewhere in the holding company system and are not available to the utility when needed for utility investment to maintain safe, adequate, and reliable service.

In addition, the holding company structure can lead to utility expenditures that support technological innovations of unregulated subsidiaries or affiliates within the corporate structure. However, the ultimate concern is that the financial viability of the utility business may be threatened by losses incurred by the non-utility businesses. For example, West Pinnacle (parent of Arizona Public Service, the largest electricity utility in Arizona) was brought close to bankruptcy due to the disastrous performance of its savings and loan subsidiary, Merabank. The FPL Group

⁵ See S & P, Standard and Poor (2007).

⁶ See Fitch (2007).

had to divest its insurance subsidiary, Colonial Penn, because of the losses it generated. Indeed, based on this experience, FPL Group subsequently decided to abandon all diversified activities.⁷

The establishment of ring-fencing requirements to avoid this contamination problem and associated increase in the cost of capital of the regulated division is currently an important policy issue in the US with the repeal of the Public Utility Holding Company Act of 1935 (PUHCA 1935). The central goal of the successor statute, the Public Utility Holding Company Act of 2005 (PUHCA 2005), is to encourage the infusion of capital into the electric and gas industries by facilitating the efficient consolidation of electricity and gas companies.⁸ Although PUCHA 2005 provides the State's Public Utility Commissions (PUC) and the Federal Energy Regulatory Commission (FERC) with access to records of holding companies and their affiliates, the repeal of PUHCA 1935 means that the federal regulatory oversight over holding companies and their affiliates has been reduced. However, some States have already established regulatory frameworks aimed at holding companies. Burns and Murphy (2006) give several examples of State ring-fencing structures. The Oregon Public Utility Commission, for instance, uses a case-by-case ring-fencing approach which is implemented through its merger review process.

Concern about the relationship among ring-fencing, diversification and risk is also present in the regulatory debate in the UK. Indeed, all the major network businesses in the energy sector have to retain an investment grade credit rating and are prevented from engaging in non-core activities.⁹

In particular, in 2005, Ofgem increased the restrictions offered by financial ring-fencing by formally adding cash lock-up provisions to licence conditions of all gas and electricity distribution businesses and announced its intention to introduce similar conditions in all other gas and electricity network licences. This mechanism is triggered when a licensee has the lowest level of credit rating consistent with investment grade (MIS Baa3, and for S&P and FitchRatings BBB-) and a credit rating agency has revised the rating outlook to negative or placed the licensee's rating on review for possible downgrade.¹⁰

In 2002, the Independent Competition and Regulatory Commission (ICRC) reviewed ring-fencing requirements for gas and electricity network service providers in the Australian Capital Territory. ICRC (2002) states that in principle any business that operates in a competitive market which is associated with, or part of, another gas and electricity business that operates in a related

⁷ See Jandik and Makhija (2005).

⁸ See Burns and Murphy (2006).

⁹ Corporate groups usually by-pass this last restriction by creating a new subsidiary.

¹⁰ See Ofwat and Ofgem (2006).

upstream or downstream regulated market should be subject to ring-fencing rules. The rules potentially apply to all businesses that operate in the market where there is a risk that competition in the market might be reduced as a result of the business relationships between the entities. The guidelines include ring-fencing measures such as legally separating the network and retail parts of their businesses; ensuring that their marketing staff and certain other operational staff are not also staff of related businesses; physically separating the offices of network providers and related businesses; separating the information systems of network providers and related businesses; developing procedures for cost allocation, shared boards of management and complaints handling, and conducting any business with related (non-retail) businesses at arm's length.

This Section strongly suggests that credit rating agencies methodologies reflect a concern with the possibility of default contamination between a regulated activity in energy markets and an unregulated business. This in turn suggests that the cost of debt of the regulated energy business might increase in the absence of ring-fencing mechanisms. This concern has been recently picked up by regulatory agencies in the US and in the UK. We label this concern the contamination problem; the idea that the revenue from the regulated energy business can be used, for instance, as collateral for investing in more risky competitive markets. In this case, the trade-off is between allowing the firm to diversify and (efficiently) invest in competitive markets and allowing rate payers to bear some of the risk of this efficient investment. Thus, this paper investigates the conditions under which it is optimal for the energy regulator to establish ring-fencing requirements in order to avoid the contamination problem.

4. The Model

We now proceed to develop a simple framework to investigate the contamination problem, that is, we will analyse how rating methodologies can affect regulated businesses and consequently regulatory policies. In our framework we examine the simple, but common, scenario where a holding energy firm has two subsidiaries. The first subsidiary operates in a regulated market such as the distribution of electricity and will be referred to as the Regulated Division. The second subsidiary operates in a competitive market such as energy trading or even the provision of internet services or airplanes leasing and will be referred to as the Unregulated Division.

The underlying characteristics of the regulated and unregulated markets follow those specified in Sappington (2003). The Regulated Division is required to satisfy the entire demand at a regulated

price p_R . The demand in the regulated market is assumed to be perfectly inelastic at output level

Q_R up to a reserve price $\bar{p}_R \geq p_R$.

The unregulated market is characterized by competition among firms that produce a homogenous product with constant returns to scale. With no participation by the Unregulated Division, competition in the unregulated market drives the market price to the marginal cost of the least-cost supplier, \bar{c}_U . The Unregulated Division is assumed to be the most efficient producer in the unregulated market due to, for instance, new technology implementation. Thus, the marginal cost of the Unregulated Division is always lower or equal to \bar{c}_U . For simplicity, consumer demand in the unregulated market is perfectly inelastic (up to a reserve price, $\bar{p}_U \geq \bar{c}_U$) at output level \bar{Q}_U . The maximum production capacity of the Unregulated Division in the unregulated market is $Q_U < \bar{Q}_U$. Thus, the profit-maximizing price for the Unregulated Division is \bar{c}_U , the marginal cost of the next most efficient producer in that market.

There is a rating agency that is responsible for setting credit ratings for both the Regulated and Unregulated Divisions. As discussed in Section 3, the actual credit agencies' rating methodologies state that the ratings are influenced by the existence of ring-fencing mechanisms separating the operations of the two divisions.

In the presence of a ring-fencing mechanism, the rating agency treats the two divisions as independent businesses. The underlying assumption is that the rating agency perceives the presence of a consistent set of ring-fencing mechanisms as a fully informative signal that the Regulated Division will not be contaminated (default risk) by its affiliate (the Unregulated Division) or its parent (the Holding). A perfectly insulated Regulated Division is seen by the rating agency as a low risk business, with low volatility of revenues.¹¹ In particular, in the presence of ring-fencing mechanisms the ratings of the Regulated and Unregulated Divisions are given by,

11 Standard rating methodology includes the analysis of both quantitative and qualitative aspects of the financial position of a firm. The former involves the analysis of specific financial ratios while the latter includes the use by the agency of its expertise on evaluating non-quantifiable factors that might influence the riskiness of firms operating in particular sectors. The analysis that follows assumes that the quantitative measures that influence each division's rating have little weight or are neutral (i.e., the current financial and economic conditions faced by both divisions are considered equivalent) and what really matters are the qualitative factors of the rating setting. That is, the subjective perception of the rating agency regarding the

respectively, ϕ_R^H and ϕ_U^L , where H stands for high and L for low, with $0 < \phi_U^L \leq \phi_R^H$, where a high credit rating is, of course, better than a low rating.

In the absence of ring-fencing, the analysis that follows assumes that the rating agency's subjective evaluation considers the existence of a transfer of risk from the Unregulated Division to the Regulated Division. It is assumed that this transfer of risk is directly proportional to the Unregulated Division's choice of output q_U . In particular, we postulate a direct relationship between q_U and the credit rating of the two divisions. First, the Regulated Division's rating is given by a continuous, decreasing function $\phi_R(q_U)$ that ranges from its highest value ϕ_R^H when $q_U = 0$ to its lowest value $\phi_R(Q_U)$. Moreover, the Unregulated Division's rating is given by the continuous, increasing function $\phi_U(q_U)$ ranging from its lowest value ϕ_U^L when $q_U = 0$ to its highest value $\phi_U(Q_U)$.

The underlying rationale for $\phi_R(q_U)$ to decrease is that q_U measures the size of the Unregulated Division and, as a consequence, the extent of the Regulated Division's exposure to its unregulated affiliate. The larger q_U , in the absence of ring-fencing, the greater the scope to leverage part of the Regulated Division's assets to support the Unregulated Division. Indeed, as seen in Section 3, Moody's rating methodology for regulated electric utilities includes the assessment of the extent of a regulated company's exposure to its unregulated businesses – the strongest credit risk position is enjoyed by a company whose business is wholly regulated.

Conversely, the underlying rationale for $\phi_U(q_U)$ to increase is due to the same fact that the larger q_U , in the absence of ring-fencing, the greater the scope for the Unregulated Division to be supported by part of the Regulated Division's assets. In this sense, in unfavourable scenarios the Unregulated Division's obligations could be offset by using the assets from its regulated affiliate while keeping its own assets safe.

It is further assumed that the credit rating function affects the marginal cost of each of the divisions. That is, an increase or decrease of risk due to a change in the credit rating of a division has a direct impact on the cost of debt and, consequently, on the marginal cost. The underlying rationale is discussed in details at the end of subsection 4.1. In particular, we can write the

overall risk faced by each business is the only differentiating factor when determining the credit ratings for

Regulated Division's marginal cost function as a continuous, increasing function $C_R(\bullet, \phi_R(q_U)) = C_R(\phi_R)$ that ranges from its lowest value $C_R(\phi_R^H) = C_R^L$ when $q_U = 0$ to its highest value $C_R(\phi_R(Q_U))$. The Unregulated Division's marginal cost function is given by a continuous, decreasing function $C_U(\bullet, \phi_U(q_U)) = C_U(\phi_U)$, which range from its highest value $C_U(\phi_U^L) = C_U^H$ when $q_U = 0$ to its lowest value $C_U(\phi_U(Q_U))$. At a basic intuitive level, the impact of a change in the credit rating on the marginal cost function will depend on the amount of debt that is rolled and/or issued.

To sum up, our framework is based on existing credit rating methodologies. In particular, according to these methodologies, in the absence of ring-fencing mechanisms the credit rating of an electricity utility is negatively affected by the company's exposure to unregulated businesses ran by an affiliate. Conversely, under the same conditions the credit rating of an unregulated business is positively affected by the existence of regulated activities ran by the electricity utility. Given this trade-off we investigate whether it is optimal for the energy regulator to establish ring-fencing mechanisms. In order to investigate and isolate the consequences of such rating practices, we do not consider other forms of risk as well as economies of scale and scope that have been addressed by the existing literature on diversification (see Section 2). Instead, given the credit agencies methodologies, our goal is to provide a simple framework to examine the impact of the holding firm's decision to invest in an unregulated market on the optimal regulation. We now examine this relationship in more detail.

4.1 The Cost Function

The postulated direct relationship between q_U and the credit rating of the two divisions is represented as follows in the absence of ring-fencing:

$$\phi_R(q_U) = \phi_R^H - \alpha q_U, \text{ where } \frac{\phi_R^H}{Q_U} > \alpha > 0 \text{ so that } \phi_R(q_U) > 0 \text{ for } q_U \leq Q_U \quad (1)$$

and

the two divisions.

$$\phi_U(q_U) = \phi_U^L + \beta q_U, \text{ where } \beta > 0 \text{ and } \phi_U(q_U) > 0 \text{ for any } q_U \quad (2)$$

In the presence of ring-fencing, $\phi_R(q_U) = \phi_R^H$ and $\phi_U(q_U) = \phi_U^L$.

It is further assumed that the credit rating function affects the marginal cost of each of the divisions as follows:

$$C_R(\phi_R(q_U)) = K + r(\phi_R(q_U))\gamma D_R \quad (3)$$

and

$$C_U(\phi_U(q_U)) = W + r(\phi_U(q_U))\delta D_U \quad (4)$$

where $K, W > 0$ are fixed costs of the regulated and unregulated divisions, respectively, and r is the cost of debt function, which is the same function for both divisions and is given by:

$$r(\phi) = \frac{1}{\phi} \quad (5)$$

Moreover, we let D_R denote the debt issued or rolled over throughout the period by the Regulated Division and γ the Regulated Division's debt amortization rate during that period. D_U denotes the debt issued or rolled over throughout the period by the Unregulated Division and δ is the Unregulated Division's debt amortization rate.

The underlying rationale for the marginal cost functions above is that firms finance investments and operating costs with both equity and debt. Moreover, short term debt is commonly used to finance operating costs and working capital requirements such as to expand a current facility, increase production and cover operating expenses. In this sense, at least a small part of the firm's debt depends upon current production and it is then a variable cost. This component of the marginal cost is assumed to vary with changes in the cost of debt following a change in the credit rating of a division.

It is worth noting that the cost of debt function has an inverse relation with the rating value. In this sense, an increase in the rating means that the division's overall risk has decreased and, as a consequence, the market cost of debt decreases.

5. Socially Optimal Ring-Fencing

This Section investigates when it is socially optimal to ring-fence the regulated business in order to avoid the contamination problem. While subsection 5.1 addresses the case where the regulator only cares about welfare in the regulated market, subsection 5.2 examines the case where the regulator cares about welfare in both regulated and unregulated markets.

5.1. Optimal Ring-Fencing under Limited Regulator's Objective Function

In this subsection, we consider a regulator who only cares about the welfare of consumers and the firm in the regulated market. First, we find the market outcomes and the associated total welfare in the regulated market in the absence of ring-fencing mechanism. In this case, the regulator solves the following maximization problem:

$$\underset{p_R}{Max} \quad \lambda Q_R \left(\bar{p}_R - p_R \right) + (1 - \lambda) \{ [p_R - C_R(\phi_R(Q_U))] Q_R \} \quad (6)$$

subject to

$$[p_R - C_R(\phi_R(Q_U))] Q_R \geq 0.$$

where λ denotes the weight assigned by the regulator to consumer surplus in the regulated market. Note that due to the unregulated market's structure, the Unregulated Division will always

produce at its maximum capacity; that is, $q_U = Q_U$.¹² In addition, the Regulated Division profit constraint is clearly binding and, therefore, the regulated price is set at marginal cost:

$$p_R = C_R(\phi_R(Q_U)) = K + r(\phi_R(Q_U))\gamma D_R.$$

Consequently, the regulator's utility is

$$\lambda Q_R \left(\bar{p}_R - C_R(\phi_R(Q_U)) \right) = \lambda Q_R \left(\bar{p}_R - (K + r(\phi_R(Q_U))\gamma D_R) \right).$$

We now consider the regulator's problem in the presence of ring-fencing mechanisms which are costless to establish. In this case the regulator solves the following problem:

$$\underset{p_R^R}{Max} \quad \lambda Q_R \left(\bar{p}_R - p_R^R \right) + (1 - \lambda) \left\{ [p_R^R - C_R^L] Q_R \right\} \quad (7)$$

subject to

$$[p_R^R - C_R^L] Q_R \geq 0.$$

Again, it is clear that the Regulated Division profit constraint is binding and that, therefore, the price in the regulated market equals the Regulated Division marginal cost: $p_R^R = C_R^L = K + r(\phi_R^H)\gamma D_R$, which now avoids the contamination problem.

Thus, in the absence of a ring-fencing structure, price in the regulated market increases by an amount equal to

$$[C_R(\phi_R(Q_U)) - C_R^L] = [r(\phi_R(Q_U)) - r(\phi_R^H)]\gamma D_R.$$

Consequently, the regulator's utility decreases by an amount equal to

¹² The unregulated division's profit is equal to $\left[\bar{c}_U - C_U(\phi_U(q_U)) \right] q_U$ and its marginal cost is a decreasing function of q_U .

$$\lambda Q_R [C_R(\phi_R(Q_U)) - C_R^L] = \lambda Q_R [r(\phi_R(Q_U)) - r(\phi_R^H)] \gamma D_R \quad (8)$$

and substituting equations (1), (2) and (5) we can rewrite (8) as follows:

$$\frac{\lambda Q_R \gamma D_R \alpha Q_U}{\phi_R^H (\phi_R^H - \alpha Q_U)} \quad (9)$$

By definition we know that $(\phi_R^H - \alpha Q_U) > 0$ so that expression (9) is greater than zero.

We have just established the following result:

Proposition 1: *When ring-fencing is costless and the regulator only considers the welfare in the regulated market, ring-fencing is always desirable.*

Moreover, from (9) above it is clear that the negative effects of diversification on the regulated market welfare are directly proportional to the initial conditions $\lambda, Q_R, \gamma, D_R, \alpha$ and Q_U . In addition, if the establishment of a ring-fencing structure creates an additional cost to the regulator or to the regulated firm (μ), the regulator should establish a ring-fencing mechanism only if $\mu \leq \frac{\lambda Q_R \gamma D_R \alpha Q_U}{\phi_R^H (\phi_R^H - \alpha Q_U)}$. Section 6 reviews the existing scant evidence on the costs of establishing ring-fencing mechanisms.

5.2. Optimal Ring-Fencing under General Regulator's Objective Function

Now, suppose that the regulator's utility function includes the total surplus in the unregulated and regulated markets and assume that ring-fencing mechanisms are absent. In this case, the regulator solves the following problem:

$$\text{Max}_{p_R} \lambda \left\{ Q_R \left(\bar{p}_R - p_R \right) + \bar{Q}_U \left(\bar{p}_U - \bar{c}_U \right) \right\} + (1-\lambda) \left\{ [p_R - C_R(\phi_R(Q_U))] Q_R + [\bar{c}_U - C_U(\phi_U(Q_U))] Q_U \right\} \quad (10)$$

subject to

$$[p_R - C_R(\phi_R(Q_U))] Q_R + [\bar{c}_U - C_U(\phi_U(Q_U))] Q_U \geq 0$$

In the same vein as in subsection 5.1, the firm's profit constraint binds. Thus, the price in the regulated market is equal to:

$$p_R = C_R(\phi_R(Q_U)) - \left[\bar{c}_U - C_U(\phi_U(Q_U)) \right] \frac{Q_U}{Q_R}$$

and

$$p_R = (K + r(\phi_R(Q_U))\gamma D_R) - \left[\bar{c}_U - (W + r(\phi_U(Q_U))\delta D_U) \right] \frac{Q_U}{Q_R}$$

and the resulting regulator's utility is equal to

$$\lambda \left\{ Q_R \left(\bar{p}_R - \left[(K + r(\phi_R(Q_U))\gamma D_R) - \left[\bar{c}_U - (W + r(\phi_U(Q_U))\delta D_U) \right] \frac{Q_U}{Q_R} \right] \right) + \bar{Q}_U \left(\bar{p}_U - \bar{c}_U \right) \right\}$$

If the regulator decides to establish a ring-fencing structure, there is no contamination between the markets. In this case the regulator has the following maximization problem:

$$\text{Max}_{p_R^R} \lambda \left\{ Q_R \left(\bar{p}_R - p_R^R \right) + \bar{Q}_U \left(\bar{p}_U - \bar{c}_U \right) \right\} + (1-\lambda) \left\{ [p_R^R - C_R^L] Q_R + [\bar{c}_U - C_U^H] Q_U \right\} \quad (11)$$

subject to

$$[p_R^R - C_R^L] Q_R + [\bar{c}_U - C_U^H] Q_U \geq 0$$

Thus, the price in the regulated market is:

$$p_R^R = C_R^L - \left[\bar{c}_U - C_U^H \right] \frac{Q_U}{Q_R} = \left(K + r(\phi_R^H) \gamma D_R \right) - \left[\bar{c}_U - (W + r(\phi_U^L) \delta D_U) \right] \frac{Q_U}{Q_R}$$

and the resulting regulator's utility is

$$\lambda \left\{ Q_R \left(\bar{p}_R - \left[\left(K + r(\phi_R^H) \gamma D_R \right) - \left[\bar{c}_U - (W + r(\phi_U^L) \delta D_U) \right] \frac{Q_U}{Q_R} \right] \right) + \bar{Q}_U \left(\bar{p}_U - \bar{c}_U \right) \right\}$$

The difference between the utility with and without ring-fencing mechanisms is $\lambda Q_R (p_R - p_R^R)$, which can be written as:

$$\lambda Q_R (p_R - p_R^R) = \lambda Q_R \left(\left[r(\phi_R(Q_U)) - r(\phi_R^H) \right] \gamma D_R + \left[r(\phi_U(Q_U)) - r(\phi_U^L) \right] \delta D_U \frac{Q_U}{Q_R} \right) \quad (12)$$

Thus, the regulator would establish a ring-fencing structure if and only if expression (12) is non-negative.

Moreover, it is easy to see that the term $\left[r(\phi_R(Q_U)) - r(\phi_R^H) \right] \gamma D_R = \left[C_R(\phi_R(Q_U)) - C_R^L \right]$ is always positive and refers to the utility diversification impact on the Regulated Division marginal cost in the absence of a ring-fencing structure. It is the cost of diversification, that is, the impact on the regulated market's outcome due to an increase in the default risk (contamination process).

Furthermore, the term $\left[r(\phi_U(Q_U)) - r(\phi_U^L) \right] \delta D_U \frac{Q_U}{Q_R} = \left[C_U(\phi_U(Q_U)) - C_U^H \right] \frac{Q_U}{Q_R}$ is always negative and refers to the utility's diversification impact on the Unregulated Division marginal cost. It is the benefit of diversification, that is, the impact on the unregulated market's outcome due to a reduction in the default risk (contamination process).

In other words, the difference between the prices depends basically on the indirect impact of Q_U on the marginal cost functions and the relative size of the unregulated market diversification by

the utility firm as measured by $\frac{Q_U}{Q_R}$. The weight of each variable is dictated by the initial market conditions. Furthermore, from (12) we know that a ring-fencing structure is preferable for the regulator if

$$\frac{r(\phi_R(Q_U)) - r(\phi_R^H)}{r(\phi_U^L) - r(\phi_U(Q_U))} \geq \sigma Q_U \quad (13)$$

where $\sigma = \frac{\delta D_U}{\gamma D_R Q_R}$.

Inequality (13) indicates that ring-fencing will be preferable if the Regulated Division's cost of debt contamination is larger than the Unregulated Division's cost of debt contamination multiplied by σQ_U . Therefore, it is easy to see that for any cost of debt function, the higher the σ the less desirable it will be for the regulator to promote a ring-fencing structure.¹³ While an increase in δ or D_U reduces the chances of ring-fencing mechanisms being desirable, an increase in γ , D_R or Q_R increases the chances of a ring-fencing structure.

Recall that δ is the Unregulated Division's debt amortization rate. An increase in δ means that the Unregulated Division will pay a larger proportion of its debt and the impact of diversification on the Unregulated Division's marginal cost becomes more pronounced and the benefit of diversification increases. Similar reasoning applies to the analysis of the effect of increasing D_U , the debt issued or rolled over throughout the period by the Unregulated Division. Moreover, an increase in γ , the Regulated Division's debt amortization rate, means that the impact of diversification on the Regulated Division's marginal cost increases because the Regulated Division will pay a larger proportion of its debt. Therefore, the cost of diversification increases. Again a similar reasoning applies to D_R , the debt issued or rolled over throughout the period by the Regulated Division. In addition and not surprisingly, an increase in Q_R increases the chances of ring-fencing mechanisms being desirable because the larger the size of the regulated market the larger will be the cost of diversification in the case of default contamination.

¹³ In the Appendix we provide a formal proof of this statement.

Inequality (13) also shows that because the cost of debt is a function of Q_U the impact of this parameter on the regulator's decision will depend on the cost of debt function. Substituting (5) in (13) we have:

$$\left(\frac{\phi_U^L \phi_U(Q_U)}{\phi_R^H \phi_R(Q_U)} \right) \left(\frac{\phi_R^H - \phi_R(Q_U)}{\phi_U(Q_U) - \phi_U^L} \right) \geq \sigma Q_U \quad (14)$$

and substituting equations (1) and (2) in (14) we have:

$$\frac{\phi_U^L (\phi_U^L + \beta Q_U)}{\phi_R^H (\phi_R^H - \alpha Q_U)} \geq \frac{\beta \sigma Q_U}{\alpha} \quad (15)$$

To address the impact of α , β and Q_U on the desirability of ring-fencing mechanisms, it is easy to see that inequality (15) defines a region R_1 in the plane (ϕ_R^H, ϕ_U^L) where $0 < \phi_U^L \leq \phi_R^H$ and $\phi_R^H > \alpha Q_U$. Thus, if R_1 increases due to a change in an initial condition, it means that the chances of ring-fencing mechanisms being desirable have increased. In the Appendix we show that if α grows then R_1 grows and consequently the desirability of ring-fencing increases. However, if β increases then R_1 decreases and the desirability of ring-fencing mechanisms also decreases. Finally, if Q_U increases then R_1 can decrease or increase depending on the other initial conditions.

An increase in α means that the derivative of ϕ_R with respect to Q_U increases in module and, as a result, the impact of Q_U in the Regulated Division's rating function becomes more significant. On the other hand, an increase in β also means that the impact of Q_U in the Unregulated Division's rating function increases but in the form of a benefit rather than a cost. This leads the regulator to avoid ring-fencing mechanisms. Finally, perhaps more surprisingly, we found that an increase in Q_U can increase or decrease the chances of ring-fencing mechanisms being desirable. In fact, while Q_U generates benefits to the Unregulated Division, it creates costs to the Regulated Division. The analysis above is summarized by the following proposition.

Proposition 2: *When the regulator's utility function includes the unregulated market as well as the regulated market the desirability of a ring-fencing structure will depend on what initial market*

conditions satisfy inequality (15). The table below summarizes the effects of changing particular initial conditions, *ceteris paribus*, on the desirability of ring-fencing mechanisms.

Increase in the Initial Condition	Effect on the Desirability of Ring-Fencing Mechanisms
δ	Reduction
D_U	Reduction
γ	Increase
D_R	Increase
α	Increase
β	Reduction
Q_R	Increase
Q_U	Reduction, Increase

6. DISCUSSION

This paper considers a situation where an energy business that operates in a regulated market such as electricity distribution diversifies into an unregulated market such as energy trading or financial services. This diversification is efficient in the sense that the firm is an efficient provider in the unregulated market. Moreover, the cost of the Unregulated Division can be reduced; for example, when the parent firm uses revenue from the regulated business as collateral to raise funds to finance operation costs in the unregulated market. We label this phenomenon the contamination problem. Contamination unambiguously reduces the cost of the Unregulated Division, but it can affect adversely the cost of the Regulated Division. The mechanism through which contamination generates adverse effects is the credit rating.

Rating agencies often examine both quantitative (financial) indicators and qualitative factors. Contamination can affect the credit rating of the Regulated Division through an agency's subjective perception of the risk faced by the Regulated Division when its future revenues are committed in investments in the riskier unregulated market. Ring-fencing can be seen in this context as an effective isolator of risk. However, ring-fencing also eliminates (or more realistically mitigates) any cost reduction in the unregulated markets.

Regulators often have requirements that regulated businesses can diversify into competitive markets as long as their credit rating is not too adversely affected (e.g., as long as the regulated

division retains an investment grading). However, regulators also often mandate ring-fencing mechanisms of varying degrees of sophistication. In this sense, the discussion on the introduction of ring-fencing seems to focus only on its benefits and does not consider its (indirect) costs.

The contribution of this paper is to show that a regulator that has a wider welfare function which includes total welfare from both regulated and unregulated markets, will not necessarily impose ring-fencing.¹⁴ In particular, we show how the desirability of ring-fencing changes as the various cost parameters change.

The analysis undertaken in this paper has also abstracted from the potentially significant direct costs of imposing ring-fencing mechanisms. These costs will vary with the stringency of ring-fencing from accounting separation to division separation to structural break up, which in our case amounts to preventing the firm from diversifying into competitive markets.

Although there are no established empirical evaluations of the direct costs imposed by ring-fencing, recent estimates from the telecommunications sector suggest that these costs can be substantial. For example, Crandall and Sidak (2002) estimate at \$40 million the direct costs of the proposed structural separation of Verizon's Pennsylvania operations. British Telecom reports that it faced a cost of £70 million for setting up its access services division BT Openreach under its voluntary division separation initiative (around 2.0% of this division annual revenues).¹⁵ Although these are estimates from more complex separation requirements in the telecommunications sector, they do suggest that imposing ring-fencing requirements is not costless.

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¹⁴ In this paper, the assumption of inelastic demand in the competitive market based on Sappington (2003) implies that all cost savings in the competitive market are captured by the firm. However, if the demand in the competitive market were perfectly elastic, then it is not difficult to see that all the benefits would be captured by consumers.

¹⁵ See <http://www.btplc.com/News/Articles/Showarticle.cfm?ArticleID=102b2b74-3c23-4df9-8438-eba2687ccea9#pdf>.

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Appendix

In this Appendix we investigate the effect of changes in α , σ , β and Q_U on the desirability of ring-fencing structures. From inequality (15) consider the region R in the plane (ϕ_R^H, ϕ_U^L) defined by:

$$R: \left(\phi_U^L + \frac{\beta Q_U}{2} \right)^2 - \left(\frac{\beta \sigma Q_U}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_U}{2} \right)^2 - \frac{\beta Q_U^2}{4} (\beta - \alpha \sigma Q_U) \geq 0$$

The boundary of R is a hyperbola, labelled H.

$$H: \left(\phi_U^L + \frac{\beta Q_U}{2} \right)^2 - \left(\frac{\beta \sigma Q_U}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_U}{2} \right)^2 - \frac{\beta Q_U^2}{4} (\beta - \alpha \sigma Q_U) = 0$$

Also, we label C the intersection of H with the first quadrant. Note that the region above C is contained in R.

It is easy to verify that the points of intersection of C with the ϕ_R^H axis are $(0,0)$ and $(\alpha Q_U, 0)$.

Recall that $0 < \phi_U^L \leq \phi_R^H$ and $\phi_R^H > \alpha Q_U$. We define the region R_1 as $R_1 = R \cap (0 < \phi_U^L \leq \phi_R^H) \cap (\phi_R^H > \alpha Q_U)$. Clearly this region is bounded by the vertical segment between the points $(\alpha Q_U, 0)$ and $(\alpha Q_U, \alpha Q_U)$, by an interval of the diagonal $\phi_R^H = \phi_U^L$ and by a piece of the hyperbola C.

The curve C can be considered as the graph of a function $\phi_U^L = \phi_U^L(\phi_R^H)$ that can be differentiated at $\phi_R^H = 0$ and $\phi_R^H = \alpha Q_U$. Thus:

$$2 \left(\phi_U^L + \frac{\beta Q_U}{2} \right) \phi_U^{L'}(\phi_R^H) - \left(\frac{2\beta\sigma Q_U}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_U}{2} \right) = 0 \quad (16)$$

At the point $\phi_R^H = 0$, $\phi_U^L = 0$ we have $\phi_U^{L'}(0) = -\sigma Q_U$ and at $\phi_R^H = \alpha Q_U$, $\phi_U^L = 0$ we have $\phi_U^{L'}(\alpha Q_U) = \sigma Q_U$.

Proof that an increase in α increases the desirability of ring-fencing:

In order to find the effect of the variation of α on the area of R_1 suppose that $\alpha_1 < \alpha_2$ and all other parameters are fixed. Then, we have two curves C:

$$C(\alpha_1): \quad \left(\phi_U^L + \frac{\beta Q_U}{2} \right)^2 - \left(\frac{\beta \sigma Q_U}{\alpha_1} \right) \left(\phi_R^H - \frac{\alpha_1 Q_U}{2} \right)^2 - \frac{\beta Q_U^2}{4} (\beta - \alpha_1 \sigma Q_U) = 0$$

$$C(\alpha_2): \quad \left(\phi_U^L + \frac{\beta Q_U}{2} \right)^2 - \left(\frac{\beta \sigma Q_U}{\alpha_2} \right) \left(\phi_R^H - \frac{\alpha_2 Q_U}{2} \right)^2 - \frac{\beta Q_U^2}{4} (\beta - \alpha_2 \sigma Q_U) = 0$$

We claim that the corresponding regions R_1 , denoted by $R_1(\alpha_1)$ and $R_1(\alpha_2)$ have the following property:

$$\text{Area of } R_1(\alpha_1) < \text{Area of } R_1(\alpha_2) \quad (17)$$

For a sufficiently large domain $\alpha Q_U < \phi_R^H \leq M$.

Let $L_r = (\phi_U^L = r \phi_R^H)$ be the line with slope r , $0 < r < 1$, passing through zero. Define:

$$p_1(r) = L_r \cap C(\alpha_1) \text{ and } p_2(r) = L_r \cap C(\alpha_2)$$

Given R_1 , to prove (17) it suffices to show that the tangent line to $C(\alpha_2)$ at $p_2(r)$ has a smaller slope than the tangent line to $C(\alpha_1)$ at $p_1(r)$. To see this, note that

$$p_i(r) = \left(\frac{\alpha_i \beta Q_U (r + \sigma Q_U)}{\beta \sigma Q_U - \alpha_i r^2}, \frac{r \alpha_i \beta Q_U (r + \sigma Q_U)}{\beta \sigma Q_U - \alpha_i r^2} \right), \quad i=1, 2$$

We can rewrite (16) as

$$\phi_U^L(\phi_R^H) = \frac{\beta\sigma Q_U \left(\phi_R^H - \frac{\alpha Q_U}{2} \right)}{\alpha \left(\phi_U^L + \frac{\beta Q_U}{2} \right)}$$

The slope of $C(\alpha_i)$ at $p_i(r)$ is:

$$\phi_U^L(\phi_R^H) = \frac{\sigma Q_U [r^2 \alpha + \beta(\sigma Q_U + 2r)]}{[\alpha(r^2 + 2r\sigma Q_U) + \beta\sigma Q_U]} = \frac{E(A\alpha_i + B)}{(C\alpha_i + D)}, \quad i=1, 2$$

Where $A = r^2$, $B = \beta(\sigma Q_U + 2r)$, $C = r^2 + 2r\sigma Q_U$, $D = \beta\sigma Q_U$ and $E = \sigma Q_U$. From this we obtain

$$\phi_U^L(p_2(r)) - \phi_U^L(p_1(r)) = \frac{E(\alpha_2 - \alpha_1)}{(C\alpha_2 + D)(C\alpha_1 + D)}(AD - BC) < 0,$$

$$\text{as } AD - BC = -2r\beta(\sigma Q_U + r)^2$$

Therefore, $\phi_U^L(p_2(r)) < \phi_U^L(p_1(r))$ as we wanted to show.

Proof that an increase in σ leads to a decrease in the desirability of ring-fencing:

In order to find the effect of the variation of σ on the area of R_1 suppose that $\sigma_1 < \sigma_2$ and all the other parameters are kept constant. Again consider the hyperbolas:

$$C(\sigma_1): \quad \left(\phi_U^L + \frac{\beta Q_U}{2} \right)^2 - \left(\frac{\beta \sigma_1 Q_U}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_U}{2} \right)^2 - \frac{\beta Q_U^2}{4} (\beta - \alpha \sigma_1 Q_U) = 0$$

$$C(\sigma_2): \quad \left(\phi_U^L + \frac{\beta Q_U}{2} \right)^2 - \left(\frac{\beta \sigma_2 Q_U}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_U}{2} \right)^2 - \frac{\beta Q_U^2}{4} (\beta - \alpha \sigma_2 Q_U) = 0$$

The hyperbolas $C(\sigma_1)$ and $C(\sigma_2)$ pass through the points $(0,0)$ and $(\alpha Q_U, 0)$. Their tangents at $(\alpha Q_U, 0)$ are $\sigma_1 Q_U$ and $\sigma_2 Q_U$, respectively. Since $\sigma_1 Q_U < \sigma_2 Q_U$, it is enough to show that $C(\sigma_1) \cap C(\sigma_2)$ is empty in the open first quadrant. This will imply that

$$\text{Area of } R_1(\sigma_2) < \text{Area of } R_1(\sigma_1)$$

The points in $C(\sigma_1) \cap C(\sigma_2)$ can be obtained as solutions of the two equations above. They imply:

$$\frac{\beta Q_U}{\alpha} \left(\phi_R^H - \frac{\alpha Q_U}{2} \right)^2 (\sigma_2 - \sigma_1) - \frac{\alpha \beta Q_U^3}{4} (\sigma_2 - \sigma_1) = 0$$

or $\left(\phi_R^H - \frac{\alpha Q_U}{2} \right)^2 = \left(\frac{\alpha Q_U}{2} \right)^2$, that is $\phi_R^H = \alpha Q_U, 0$. This means that the intersection $C(\sigma_1) \cap C(\sigma_2)$ consists of two points: $(0,0)$ and $(\alpha Q_U, 0)$.

Thus, $C(\sigma_1) \cap C(\sigma_2)$ is empty in the open first quadrant as we wanted to demonstrate.

Proof that as β increases, the desirability of ring-fencing decreases:

In order to find the effect of the variation of β on the area of R_1 we can observe below that β appears in the second derivative of $\phi_U^L(\phi_R^H)$ at the point $(\alpha Q_U, 0)$. In fact, it is easy to see that

$$\left(\phi_U^L + \frac{\beta Q_U}{2} \right) \phi_U^{L''}(\phi_R^H) + \left(\phi_U^L(\phi_R^H) \right)^2 - \left(\frac{\beta \sigma Q_U}{\alpha} \right) = 0$$

At the point $\phi_R^H = \alpha Q_U, \phi_U^L = 0$ we have $\left(\frac{\beta Q_U}{2}\right) \phi_U^{L''}(\alpha Q_U) = \left(\frac{\beta \sigma Q_U}{\alpha}\right) - (\sigma Q_U)^2$ and

$$\phi_U^{L''}(\alpha Q_U) = 2\sigma \left(\frac{1}{\alpha} - \frac{\sigma Q_U}{\beta} \right).$$

By the Taylor Theorem we have that for ϕ_R^H near αQ_U :

$$\phi_U^L(\phi_R^H) = \phi_U^L(\alpha Q_U) + \phi_U^{L'}(\alpha Q_U)(\phi_R^H - \alpha Q_U) + \phi_U^{L''}(\alpha Q_U) \frac{(\phi_R^H - \alpha Q_U)^2}{2} + \text{Residuals}$$

Suppose that $\beta_1 < \beta_2$ and all other variables are kept constant. Then, we have the following two curves $C(\beta_1)$ and $C(\beta_2)$:

$$C(\beta_1): \quad \left(\phi_U^L + \frac{\beta_1 Q_U}{2} \right)^2 - \left(\frac{\beta_1 \sigma Q_U}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_U}{2} \right)^2 - \frac{\beta_1 Q_U^2}{4} (\beta_1 - \alpha \sigma Q_U) = 0$$

$$C(\beta_2): \quad \left(\phi_U^L + \frac{\beta_2 Q_U}{2} \right)^2 - \left(\frac{\beta_2 \sigma Q_U}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_U}{2} \right)^2 - \frac{\beta_2 Q_U^2}{4} (\beta_2 - \alpha \sigma Q_U) = 0$$

We proceed now to prove that $C(\beta_1)$ and $C(\beta_2)$ do not intersect in the open first quadrant. When β varies, all other variables remain constant, the intersection $C(\beta_1) \cap C(\beta_2)$ is obtained as solution of the following system:

$$\left(\phi_U^L \right)^2 + \beta_1 Q_U \phi_U^L - \left(\frac{\beta_1 \sigma Q_U}{\alpha} \right) \left(\left(\phi_R^H \right)^2 - \alpha Q_U \phi_R^H \right) = 0$$

$$\left(\phi_U^L \right)^2 + \beta_2 Q_U \phi_U^L - \left(\frac{\beta_2 \sigma Q_U}{\alpha} \right) \left(\left(\phi_R^H \right)^2 - \alpha Q_U \phi_R^H \right) = 0$$

From this we obtain

$$\beta_2 \left(\left(\phi_U^L \right)^2 + \beta_1 Q_U \phi_U^L \right) - \beta_1 \left(\left(\phi_U^L \right)^2 + \beta_2 Q_U \phi_U^L \right) = 0$$

Thus, $\phi_U^L = 0$ and $(\beta_2 - \beta_1)\phi_U^L = 0$, that is, $\phi_U^L = 0$ again. This means that the intersection $C(\beta_1) \cap C(\beta_2)$ consists of two points: $(0,0)$ and $(\alpha Q_U, 0)$. Consequently, $C(\beta_1) \cap C(\beta_2)$ is empty in the open first quadrant.

In addition, near αQ_U we have two curves T_1 and T_2 that have the following expressions:

$$T_1: \quad \phi_U^L(\phi_R^H) = \sigma Q_U (\phi_R^H - \alpha Q_U) + 2\sigma \left(\frac{1}{\alpha} - \frac{\sigma Q_U}{\beta_1} \right) \frac{(\phi_R^H - \alpha Q_U)^2}{2} + \text{Re siduals}$$

$$T_2: \quad \phi_U^L(\phi_R^H) = \sigma Q_U (\phi_R^H - \alpha Q_U) + 2\sigma \left(\frac{1}{\alpha} - \frac{\sigma Q_U}{\beta_2} \right) \frac{(\phi_R^H - \alpha Q_U)^2}{2} + \text{Re siduals}$$

$$\text{Also, we know that } \left[\phi_U^L(\alpha Q_U) \right]_1 = 2\sigma \left(\frac{1}{\alpha} - \frac{\sigma Q_U}{\beta_1} \right) < 2\sigma \left(\frac{1}{\alpha} - \frac{\sigma Q_U}{\beta_2} \right) = \left[\phi_U^L(\alpha Q_U) \right]_2$$

Therefore, for ϕ_R^H near αQ_U :

$$\left[\phi_U^L(\phi_R^H) \right]_1 < \left[\phi_U^L(\phi_R^H) \right]_2$$

Since $C(\beta_1)$ and $C(\beta_2)$ do not intersect in the open first quadrant, then:

$$\text{Area of } R_1(\beta_2) < \text{Area of } R_1(\beta_1)$$

Therefore, if β increases then the area of R_1 decreases.

Proof that the impact of an increase in Q_U on the desirability of ring-fencing is dependent on the market initial conditions:

In order to find the effect of the variation of Q_U on the area of R_1 suppose that $Q_{U1} < Q_{U2}$ and all the other parameters are kept constant. Again consider the hyperbolas:

$$C(Q_{U1}): \quad \left(\phi_U^L + \frac{\beta Q_{U1}}{2} \right)^2 - \left(\frac{\beta \sigma Q_{U1}}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_{U1}}{2} \right)^2 - \frac{\beta Q_{U1}^2}{4} (\beta - \alpha \sigma Q_{U1}) = 0$$

$$C(Q_{U2}): \quad \left(\phi_U^L + \frac{\beta Q_{U2}}{2} \right)^2 - \left(\frac{\beta \sigma Q_{U2}}{\alpha} \right) \left(\phi_R^H - \frac{\alpha Q_{U2}}{2} \right)^2 - \frac{\beta Q_{U2}^2}{4} (\beta - \alpha \sigma Q_{U2}) = 0$$

Writing $(\alpha^*, \beta^*, \sigma^*) = \frac{Q_{U2}}{Q_{U1}} (\alpha, \beta, \sigma)$ we can transform $C(Q_{U1})$ obtaining

$C(\alpha^*, \beta^*, \sigma^*) = C(Q_{U2})$. This means that a change in Q_U from Q_{U1} to Q_{U2} is equivalent to a change in α , σ and β by a factor $\frac{Q_{U2}}{Q_{U1}}$, keeping Q_U fixed.

Therefore, we can state the following: If the Area of $R_1(\alpha, \beta, \sigma, Q_{U1}) <$ Area of $R_1(\alpha^*, \beta^*, \sigma^*, Q_{U1})$ then the Area of $R_1(Q_{U1}) <$ Area of $R_1(Q_{U2})$. In this case when Q_U grows then R_1 grows and consequently the desirability of ring-fencing increases.